IV. *Ether* is found to have the same action on the amylolysis in the excised liver as chloroform, but to a much less marked extent. Its action in bringing about structural changes in the liver cells is also less marked.

V. *Pyrogallic acid*, in 0·25 per cent. neutral solution, acts in the same manner upon the process of amylolysis and upon the liver cells.

VI. *Morphin* (0·005 to 0·025 per cent.), *curare*, *nitrate of amyl* (vapour through salt solution), *and salicylate of soda* (0·5 per cent.), neither increase hepatic amylolysis nor do they accelerate the cellular changes. The glycosuria caused by the administration of the first three of these is not due to increased hepatic amylolysis.

The products of hepatic amylolysis in the early and in the later stage were also investigated. In the early stage, glucose appears to be formed directly, and no intermediate bodies, such as dextrins or maltose, occur. In the later amylolysis, the former of these, possibly the latter, are always found.

In the light of these observations, the nature of the hepatic amylolysis is considered, and it is maintained that the evidence shows that the early rapid amylolysis is different from the later slow process, and that it is simply the result of the katabolic changes in the protoplasm accentuated as death occurs ; that it is, in fact, simply an exaggeration of the process of amylolysis during life ; and that it is in no way due to the action of a zymin, but is comparable to the production of mucin from mucinogen, and zymin from zymogen.

The later slow amylolysis is next considered. The development of an acid reaction, partly, at least, due to lactic acid, is shown ; but experiments are given indicating that the acidity is not the cause of the amylolysis. The influence of micro-organisms is also investigated, and experiments are given to show that the process goes on as rapidly when these are excluded as when they are present. The evidence of the existence of a zymin in the dead liver is considered, and the conclusion is drawn that the later slow amylolysis is due to the action of such a zymin, probably developed during the disintegration of the liver cells.

II. " On certain Correlated Variations in *Carcinus mœnas*." By W. F. R. WELDON, M.A., F.R.S., Fellow of St. John's College, Cambridge, Professor of Zoology in University College, London.   Received August 9, 1893.

In previous communications I have discussed the variations in size occurring in one or two organs of the common shrimp (*Crangon vulgaris*). In these papers it has been shown (1) that the observed

deviations from the average size of every organ measured are grouped symmetrically about the average, and occur with a frequency corresponding closely to that indicated by the probability integral; and (2) that the " degree of correlation " between a given pair of organs is approximately the same in each of five local races of the species (' Roy. Soc. Proc.,' vol. 47, p. 445, and vol. 51, p. 2). In what follows I shall describe the results obtained by measuring certain parts of the shore crab (*Carcinus mœnas*) in two samples, one from the Bay of Naples, and one from Plymouth Sound, each sample consisting of 1,000 adult females.

1.—*The Variation of Individual Organs.*

The measurements made were as follows :—

1. *The total length of the carapace* (fig. 1, AB), in a straight line from the tip of the median inter-orbital tooth to the middle of the posterior margin.
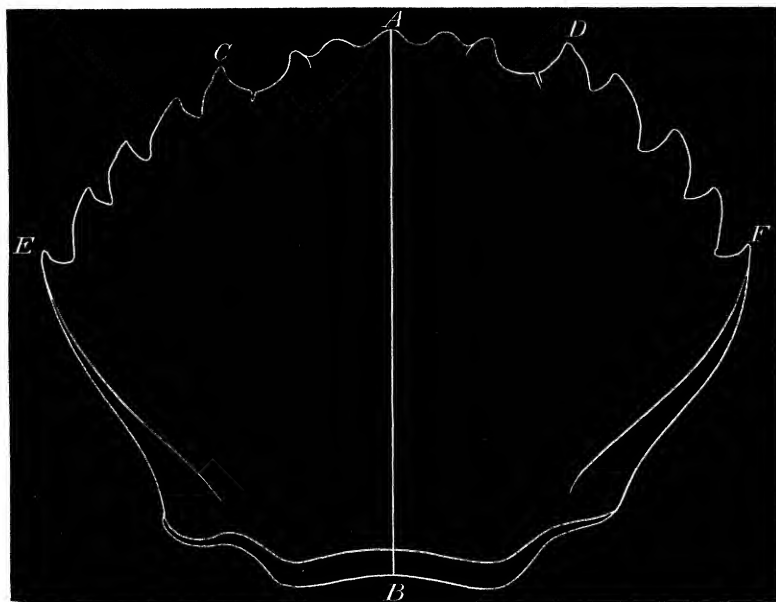

Fig. 1.—Diagram to show the parts of the carapace measured. The diagram is drawn to scale, the right half representing a perfectly average Plymouth crab, the left an average crab from Naples.

2. *The total breadth* of the carapace, in a straight line from tip to tip of the posterior lateral teeth (fig. 1, EF).

3. *The frontal breadth*, from tip to tip of the anterior lateral teeth (fig. 1, CD).

4. *The right antero-lateral margin*, from the tip of the median inter-orbital tooth to the tip of the postero-lateral tooth (fig. 1, AF).

5. *The right dentary margin*, measured in a straight line from the tip of the antero-lateral to the tip of the postero-lateral tooth (fig 1, DF).

6. *The left antero-lateral margin*, measured in the same way as the right.

7. *The left dentary margin*.

8. *The sternal breadth*, measured between the articulations of the great chelæ.

9. *The meropodite of the right chela*, measured in a straight line between the inner articulations.

10. *The carpopodite of the right chela*, from the inner articulation, in a straight line to the tip.

11. *The proximal portion of the same carpopodite*, in a straight line from the inner articulation to the tip of the inner spine, at the base of the dactylopodite.

The dimensions 2—11 were expressed in terms of the total length of the carapace taken as 1000; and, in order to reduce the effect of possible errors of measurement, the values so obtained were grouped together in fours, the groups being so selected that no two individuals in any one of them differed by more than 0·004 of the carapace length.

As an example of the way in which the values thus obtained were distributed, the measurements of the right antero-lateral margin in Naples and in Plymouth may be examined. The results of these measurements are shown in Tables I and II. The frequency with which every observed magnitude of this portion of the carapace occurred in the Naples specimens is given in the second column of Table I. The arithmetic mean of all these values is 752·22 thousandths of the carapace length; and the observations will be seen to cluster with a fair degree of symmetry around this value, the symmetry of distribution being, perhaps, more readily seen by the eye in the diagram, fig. 2. The total number of individuals in the sample was 999, and of these 513 had the antero-posterior margin greater than the average size, 486 having this portion of the carapace below the average. The arithmetic mean of all the deviations from the average, or "mean error" of distribution, was found to be 8·71 units; and the modulus is therefore $8·71 \times 1·77 = 15·42$ units. A probability curve, with modulus $= 15·42$ units, has been drawn by a dotted line in fig. 2; and the close agreement between this curve and the observed curve of distribution, which is indicated by a thick line, is very striking. In order to make a more accurate comparison possible, the number of individuals corresponding to each observed magnitude, on

the hypothesis that this probability curve represents the real distri-
bution about the mean, has been calculated from the tables of the
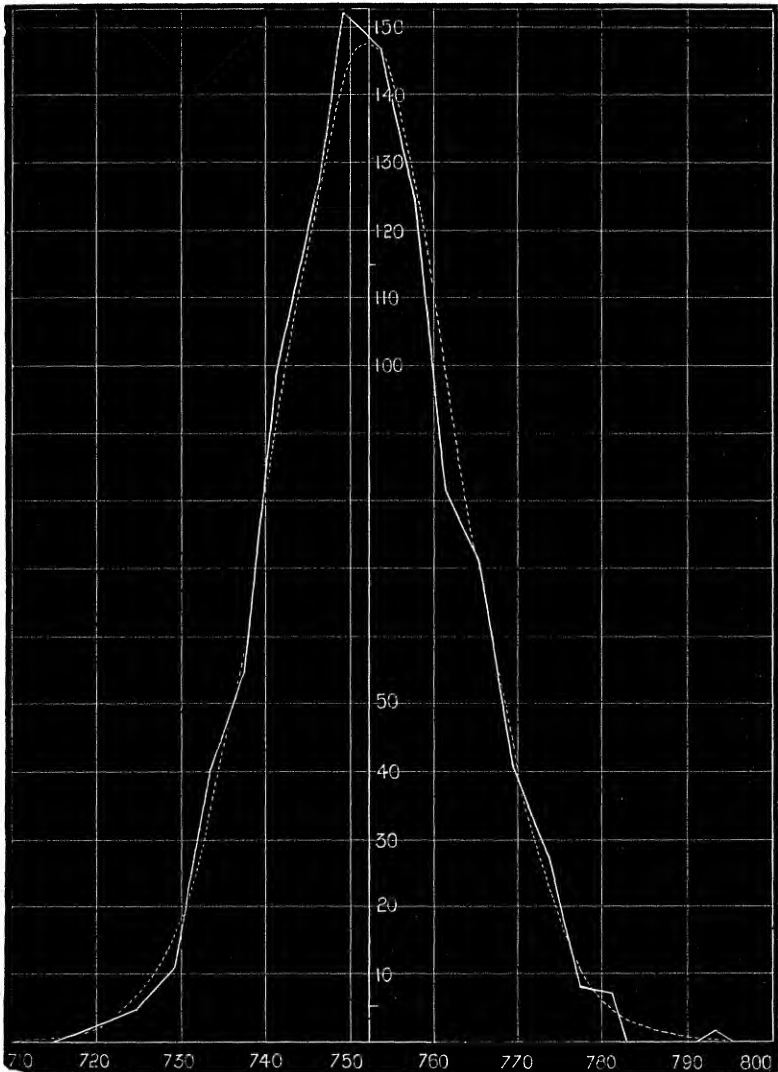probability integral, and is given in the third column of Table I.   In

FIG. 2.—Diagram showing the frequency of occurrence of all observed lengths
of the antero-lateral margin of the carapace in 999 female crabs from Naples.
The abscissa scale represents thousandths of the total carapace length. The vertical
scale represents numbers of individuals.

spite of some considerable discrepancies, the general agreement
between the second and third columns of the table is undeniable.

Table I.—Distribution of Lengths of Antero-lateral Margin of Cara-
pace in 999 Female *Carcinus mœnas* from Naples.

Dimension in thousandths of carapace length.	Number of individuals observed.	Number calculated from probability integral ($c = 15\cdot42$).
792—795	2	
788—791	0	
784—787	0	16·4
780—783	7	
776—779	8	
772—775	28	22·2
768—771	41	42·1
764—767	72	70·2
760—763	82	102·0
756—759	126	130·0
752—755	147	144·8
748—751	152	141·1
744—747	121	120·3
740—743	98	87·0
736—739	55	59·6
732—735	40	34·3
728—731	11	16·7
724—727	5	
720—723	3	11·8
716—719	1	

The right antero-lateral margin of the Plymouth individuals, when
treated in a similar way, gave the following results :—

Arithmetic mean . . . . .   762·70 thousandths.
Mean error. . . . .   9·77   ,,
Modulus . . . .   17·29   ,,

The frequency with which individual deviations from the average
occur is compared with that indicated by a probability equation of
the appropriate modulus in Table II.

These two examples will give a fair idea of the extent to which
the distribution of the observed magnitudes of each organ about the
mean of all of them corresponds to that indicated by the probability
equation.   A similar treatment of every other set of measures would
serve no useful purpose; it will be sufficient to give, in the following
table, the mean value, and the probable error of distribution about
that value, of every organ measured.   The probable error is given
below, instead of the mean error, because it is the constant which has

Table II.—Distribution of lengths of Antero-lateral Margin of Carapace in 999 Female *Carcinus* from Plymouth.

Length in thousandths of carapace length.	Number of individuals observed.	Numbers indicated by probability integral.
796—799	6	⎫
792—795	3	⎬ 21·5
788—791	7	⎭
784—787	19	23·2
780—783	44	45·2
776—779	70	61·6
772—775	94	87·5
768—771	101	110·5
764—767	140	125·6
760—763	128	134·0
756—759	105	112·7
752—755	100	97·2
748—751	79	62·2
744—747	47	53·4
740—743	25	29·2
736—739	20	15·8
732—735	4	⎫
728—731	6	⎬ 13·2
692—695	1	⎭

the smallest numerical value of any in general use. This property renders the probable error more convenient than either the mean error, the modulus, or the error of mean squares, in the determination of the degree of correlation which will be described below.

Table III.—Mean Length and Probable Error of Various Parts of *Carcinus mœnas* in Naples and Plymouth, in thousandths of the Carapace length.

Organ.	Naples.		Plymouth.	
	Mean.	Probable error.	Mean.	Probable error.
Total breadth of carapace..	1261·98	13·44	1292·74	13·84
Frontal breadth of carapace	(see below)		604·94	9·96
R. antero-lateral margin...	752·22	7·26	762·70	8·24
L. ,, ,, ...	744·98	7·26	758·54	8·20
R. dentary margin	463·86	8·60	495·14	9·28
L. ,, ,, ...	457·74	9·32	491·86	9·44
Sternal breadth...........	434·66	6·44	442·30	6·52
Meropodite R. chela	262·38	5·88	279·74	5·36
Carpopodite R. chela......	597·22	21·68	608·90	15·32
Proximal portion R. chela..	395·46	22·80	414·38	14·00

The only case in which an undoubtedly asymmetrical result was
obtained is that of the frontal breadth of the Naples specimens.
From an inspection of the curve of distribution of these magnitudes,
I was led to hope that the result obtained might arise from the
presence, in the sample measured, of two races of individuals,
clustered symmetrically about separate mean magnitudes.   Professor
Karl Pearson has been kind enough to test this supposition for me :
he finds that the observed distribution corresponds fairly well with
that resulting from the grouping of two series of individuals, one
with a mean frontal breadth of 630·62 thousandths, and a probable
error of 12·06 thousandths ; the other with a mean breadth of
654·66 thousandths, and a probable error of 8·41 thousandths.   Of the
first race, Professor Pearson's calculation gives 414·5 individuals, of
the second, 585·5.   The degree of accuracy with which this hypo-
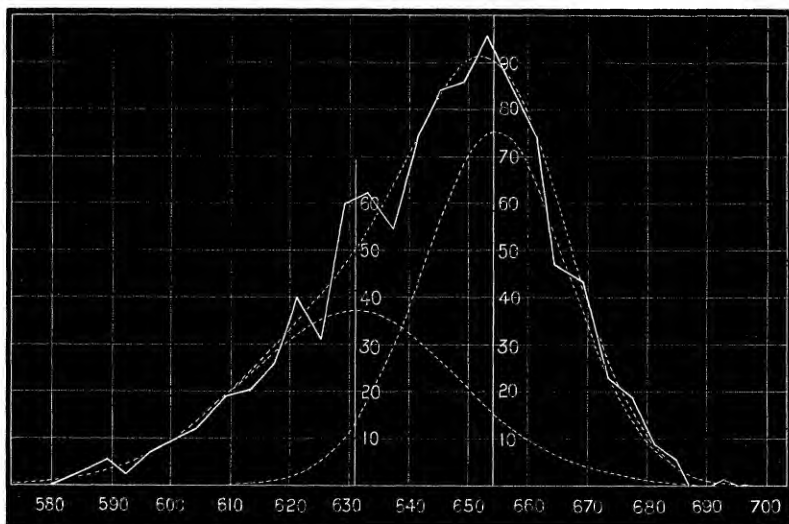thesis fits the observations may be gathered from fig. 3.

Fɪɢ. 3.—Diagram to show the distribution of all observed frontal breadths in
the Naples specimens.   The horizontal scale represents thousandths of the cara-
pace length, the vertical scale numbers of individuals.   Each ordinate of the upper
dotted curve is the sum of the corresponding ordinates of the two component
curves.

We may, therefore, assume that the female *Carcinus mœnas* is
slightly dimorphic in Naples with respect to its frontal breadth ; and
that the individuals belonging to the two types are distributed in the
proportion of nearly two to three.

2.—*The Correlation of Pairs of Organs : Galton's Function.*

The method adopted to determine the degree of correlation between two organs was that proposed by Mr. Galton ('Roy. Soc. Proc.,' vol. 40, p. 63). The measures obtained were sorted into groups, such that in each group the deviation X of an organ A from its average was constant. The mean deviation from its average of a second organ B was determined in each of these groups. Calling y_m the mean deviation of B, the ratio y_m/X was found to be approximately the same for all values of X. The same individuals were then sorted into groups in each of which the deviation Y of the organ B was constant : and in each of these groups, x_m, the mean deviation of A, was determined. As before, the ratio x_m/Y was approximately constant.

Mr. Galton has shown that if Q_a, Q_b be the probable errors of the organs A and B respectively, then

$$\frac{y_m/Q_b}{X/Q_a} = \frac{x_m/Q_a}{Y/Q_b} = r, \text{ a constant.}$$

The constant here denoted by r is evidently a measure of the degree to which abnormality in one organ is accompanied by abnormality in a second. It becomes ± 1 when a change in one organ involves an equal change in the other, and 0 when the two organs are quite independent. The importance of this constant in all attempts to deal with the problems of animal variation was first pointed out by Mr. Galton in the paper already referred to : and I would suggest that the constant whose changes he has investigated, and whose importance he has indicated, may fitly be known as " Galton's function."

As an example of the mode of determining this function, the correlation between the right and left antero-lateral margins of the Naples crabs may be examined, by means of the data given in the following tables.

In Table IV the measurements of the right antero-lateral margin have been sorted into groups, each group containing individuals which differ by not more than 0·004 of the carapace length, the magnitudes included in each group being given in the first column. In each of these groups the mean size of the left antero-lateral margin was determined, and the value obtained is given in the second column. From these data, and from the reciprocal data of the next table, the mean value of Galton's function was found to be 0·76, and the extent to which this value fits the individual cases may be estimated from the third and fourth columns of the table. In the third column, the value of the left antero-posterior margin corresponding to every value of the right margin

Table IV.—Mean Value of Left Antero-lateral Margin (*la*) for every Observed Value of the Right Antero-lateral Margin (*ra*) in 1000 Female *Carcinus* from Naples.

$M_{la} = 744\cdot98$; $Q_{la} = 7\cdot26$. $M_{ra} = 752\cdot22$; $Q_{ra} = 7\cdot26$.

Length of *ra*.	Mean associated length of *la*.	Length of *la* when $r = 0\cdot76$.	Difference.
Over 775	762·85	763·01	− 0·16
772—775	761·66	761·15	+ 0·51
768—771	757·78	758·11	− 0·33
764—767	754·78	755·07	− 0·29
760—763	750·30	752·03	− 1·73
756—759	749·26	748·99	+ 0·27
752—755	746·66	745·95	+ 0·71
748—751	742·50	742·91	− 0·41
744—747	739·94	739·87	+ 0·07
740—743	737·66	736·83	+ 0·83
736—739	733·78	733·79	− 0·01
732—735	729·58	730·75	− 1·17
Under 732	729·50	726·19	+ 3·31

has been calculated on the assumption that $r = 0\cdot76$; and in the fourth column the difference between this value and that actually observed has been recorded. It will be admitted that these differences are in all cases small, being in every case less than four thousandths of the carapace length, that is to say, less than one unit of the measures employed.

The reciprocal result, obtained by sorting the individuals into groups in which the left antero-lateral margin had a constant value, and observing the mean associated value of the right margin, is shown in Table V, which will be understood without further explanation. In this table also the differences between the observed and the calculated values are small, being always less than 0·002 of the carapace length, or half a unit of measurement.

A similar examination of the right and left antero-lateral margins of the Plymouth specimens gave 0·78 as the mean value of Galton's function : and, considering the roughness of the method employed, and the small number of individuals examined, these two values may be considered identical.

The approach to identity between the corresponding values of this function in Naples and Plymouth is generally close, and may be gathered from Table VI.

The numbers given in this table show a remarkable degree of coincidence between the values of r derived from an investigation of the same pair of organs in the two races examined. There are, in some cases, considerable differences between the value in Naples and

Table V.—Mean Value of Right Antero-lateral Margin (*ra*) for every observed Value of Left Antero-lateral Margin (*la*) in 1000 Female *Carcinus* from Naples.

Length of *la*.	Mean associated length of *ra*.	Length of *ra* when *r* = 0·76.	Difference.
Over 767	773·10	772·68	+ 0·42
764—767	767·22	767·82	− 0·60
760—763	766·10	764·78	+ 1·32
756—759	761·90	761·74	+ 0·16
752—755	758·18	758·70	− 0·52
748—751	755·26	755·66	− 0·40
744—747	751·46	752·62	− 1·16
740—743	749·46	749·58	− 0·12
736—739	747·58	746·54	+ 1·04
732—735	744·90	743·50	+ 1·40
728—731	740·50	740·46	+ 0·04
724—727	737·06	737·42	− 0·36
Under 724	734·78	734·38	+ 0·40

Table VI.—Values of Galton's Function for corresponding Pairs of Organs in 1000 Female *Carcinus* from Plymouth and 1000 from Naples.

Pairs of organs.	Naples.	Plymouth.
Total carapace breadth and frontal breadth............	0·08	0·10
„ „ R. antero-lateral margin	0·66	0·65
„ „ L. „ „	0·62	0·65
„ „ R. dentary margin	0·50	0·55
„ „ L. „	0·47	0·51
„ „ sternal breadth............	0·15	0·15
Breadth frontal and R. antero-lateral margin...........	0·29	0·24
„ „ L. „ „	0·27	0·22
„ „ R. dentary margin	−0·23	−0·18* (−0·24)
„ „ L. „	−0·26	−0·20* (−0·24)
„ „ sternal breadth	0·27	0·24
R. antero-lateral margin and L. antero-lateral margin	0·76	0·78
„ „ R. dentary margin........	0·71	0·78
„ „ L. „	0·60	0·70
„ „ sternal breadth............	0·15	0·12
R. dentary margin and L. antero-lateral margin.........	0·63	0·67
„ „ L. dentary margin	0·82	0·83
„ „ sternal breadth................	−0·03	−0·01
L. antero-lateral margin and L. dentary margin..	0·76	0·80
„ „ sternal breadth............	0·15	0·18
L. dentary margin and sternal breadth................	−0·04	+0·06
R. meropodite and R. carpopodite.....................	0·48	0·45
„ proximal portion of carpus	0·43	0·45

* These determinations contain one or two quite erratic values, by omitting which the result in brackets is obtained.

the value in Plymouth, as shown in the table; but these differences
are in no case great enough to justify the assertion that the degree of
correlation is really different in the two cases.   As an example of the
small importance of these differences, Table VII may be of use.   In
this table the mean values of the left dentary margin for every
observed value of the right antero-lateral margin are given in the
first two columns.   The value of Galton's function for this pair of
organs is given above as 0·70 in Plymouth, 0·60 in Naples.   In the
third column of Table VII, the mean value of the left dentary margin
has been calculated for a value of $r = 0·65$, and the results are com-
pared with those obtained by observation.

Table VII.—Mean Value of Left Dentary Margin (*ld*) for every ob-
served Value of the Right Antero-lateral Margin (*ra*) in 1000
Female *Carcinus* from Plymouth.

$$M_{ld} = 491·86 \; ; \; Q_{ld} = 9·44. \quad M_{ra} = 762·70 \; ; \; Q_{ra} = 8·24.$$

Length of *ra*.	Mean associated length of *ld*.	Length of *ld* when $r = 0·65$.	Difference.
Over 783	511·50	511·74	− 0·24
780—783	507·50	505·87	+ 1·73
776—779	502·70	502·89	− 0·19
772—775	499·38	499·91	− 0·53
768—771	496·64	496·93	− 0·29
764—767	495·02	493·95	+ 1·07
760—763	491·22	490·97	+ 0·25
756—759	486·98	487·99	− 1·01
752—755	484·62	485·01	− 0·39
748—751	483·18	482·03	+ 1·15
744—747	476·10	479·05	− 2·95
740—743	474·06	476·07	− 2·01
736—739	467·90	473·09	− 4·81
Under 736	472·22	465·58	+ 6·64

With the exception of the last two values, the degree of coincidence
between the calculated and observed values is close enough to be of
value as an indication of the manner in which the variations are
distributed : and even in the case of the last two values the difference
is only 1·5 times the unit of measurement.

It may, therefore, be asserted that the investigation which has
been described *does not demonstrate a difference* between the value of
Galton's function for a given pair of organs in Naples and the corre-
sponding value in Plymouth.   The values obtained are not in all
cases shown to be identical, but the differences between them are
within the limits of error of the method employed; and in the worst
case it has been shown that the errors arising from a neglect of the

observed discrepancy between two corresponding values are not of a very serious kind. So that in any discussion of the variation of the twenty-three pairs of organs discussed in the present paper, or of the pairs of shrimp organs discussed in my previous communication, it may be assumed as at least an empirical working rule that Galton's function has the same value in all local races.   The question whether this empirical rule is rigidly true will have to be determined by fuller investigation, based on larger samples: but the value of a merely empirical expression for the relation between abnormality of one organ and that of another is very great.   It cannot be too strongly urged that the problem of animal evolution is essentially a statistical problem : that before we can properly estimate the changes at present going on in a race or species we must know accurately (*a*) the percentage of animals which exhibit a given amount of abnormality with regard to a particular character; (*b*) the degree of abnormality of other organs which accompanies a given abnormality of one; (*c*) the difference between the death rate per cent. in animals of different degrees of abnormality with respect to any organ ; (*d*) the abnormality of off-spring in terms of the abnormality of parents, and *vice versâ*.   These are all questions of arithmetic ; and when we know the numerical answers to these questions for a number of species we shall know the direction and the rate of change in these species at the present day—a knowledge which is the only legitimate basis for speculations as to their past history and future fate.

III. "Contributions to the Mathematical Theory of Evolution." By Karl Pearson, M.A., Professor of Applied Mathematics, University College.   Communicated by Professor Henrici, F.R.S.   Received October 18, 1893.

(Abstract.)

1. If a series of measurements, physical, biological, anthropological, or economical, not of the same object, but of a group of objects of the same type or family, be made, and a curve be constructed by plotting up the number of times the measurements fall within a given small unit of range to the range, this curve may be termed a *frequency curve*.   As a rule this frequency curve takes the well known form of the curve of errors, and such a curve may be termed a *normal frequency curve*.   The latter curve is symmetrical about its maximum ordinate.   Occasionally, however, frequency curves do not take the normal form, and are then generally, but not necessarily, asymmetrical.   Such abnormal curves arise particularly in biological measurements ; they have been found by Professor
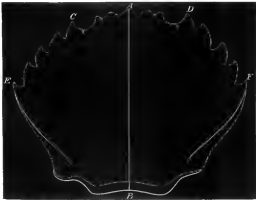
FIG. 1.—Diagram to show the parts of the carapace measured. The diagram is drawn to scale, the right half representing a perfectly average Plymouth crab, the left an average crab from Naples.
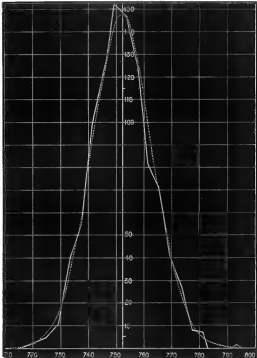
FIG. 2.—Diagram showing the frequency of occurrence of all observed lengths of the antero-lateral margin of the carapace in 999 female crabs from Naples. The abscissa scale represents thousandths of the total carapace length. The vertical scale represents numbers of individuals.
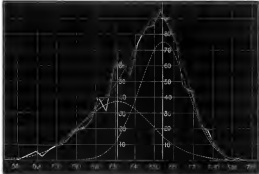
Fig. 8.—Diagram to show the distribution of all cherred frontal breadths in the Naples specimens. The horizontal scale represents thousandths of the compact length, the vertical scale numbers of individuals. Each ordinate of the upper dotted curve is the sum of the corresponding ordinates of the two component curves.